* There was no RA 23 *

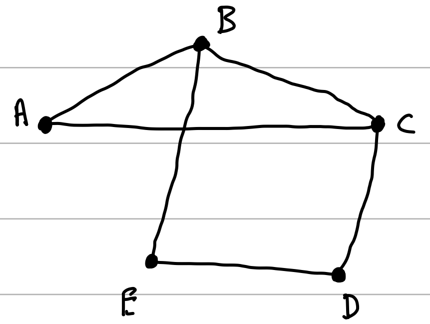
RA 24

13.1 Introduction to Graphs

- In an undirected graph, the edges are unordered pairs of vertices, which are useful for modeling relationships that are symmetric.

- Parallel edges are multiple edges between the same pair of vertices.

- A graph that does not have any parallel edges or self loops is called a simple graph.



- If there is an edge between two vertices, the vertices are adjacent.
- Vertices b and e are endpoints of edge $\{b, e\}$. The edge $\{b, e\}$ is incident to vertices b and e
- A vertex c is a neighbor of vertex b i.f.f. $\{b, c\}$ is an edge.
- In a simple graph, the degree of a vertex is the number of neighbors it has.
- The total degree of a graph is the sum of the degrees of all the vertices.
- In a d-regular graph, all vertices have degree d.

- Let $G=(V,E)$ be an undirected graph. Then twice the number of edges is equal to the total degree.

$$\sum_{v \in V} \deg(v) = 2 \cdot |E|$$

- K_n is called the <u>complete graph</u> on n vertices. K_n has an edge between every pair of vertices.
  ↳ K_n is sometimes called a <u>clique</u> of size n.

- C_n is called a cycle on n vertices. The edges connect the vertices in a ring.

- $K_{n,m}$ has n+m vertices. The vertices are divided into 2 sets: one with m vertices and one set with n vertices. There are no edges between vertices in the same set, but there is an edge between every vertex in one set and every vertex in the other set.

13.3 Graph Isomorphism

- Two graphs are isomorphic if there is a correspondance between the vertex sets of each graph such that there is an edge between two vertices of one graph i.f.f. there is an edge between the corresponding vertices of the second graph.

- A property <u>preserved under isomorphism</u> if whenever two graphs are isomorphic, one graph has the property i.f.f. the other graph also has the property.

- The <u>degree sequence</u> of a graph is a list of the degrees of all of the vertices in non-increasing order.

  - The degree sequence of a graph is preserved under isomorphism.

13.8 Planar Graphs

- An *embedding* for $G = (V, E)$ is an assignment of the vertices to points in the plane and an assignment of each edge to a continuous curve.

- An embedding is a *planar embedding* if none of the edges cross.

- There is a *crossing* between 2 edges in an embedding if their curves intersect at a point that is not a common endpoint.

- A graph G is a *planar graph* if the graph has a planar embedding.

- In a planar embedding there is always an infinite region called the *exterior region*.

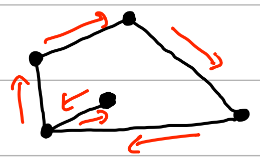
- The *complement of an embedding* is the set of all points in the plane that are not a vertex or part of a curve corresponding to an edge.

- A *region* is a set of points in the complement of an embedding that forms a maximal continuous set.

- *Euler's Identity*
  - Consider a planar embedding of a connected graph G. Let n be the number of vertices in G, m the number of edges, and r the number of regions in the embedding. Then: $n - m + r = 2$

- *Degree of a region*: the number of edges traversed around a region. (one edge can count twice)

$\boxed{\text{Ex}}$



} Degree = 6

- Number of edges in a planar graph:
  - Let G be a connected planar graph. Let n be the number of vertices in G and m the number of edges. If $n \geq 3$, then $m \leq 3n-6$